

Calibration errors unleashed: effects on cosmological parameters and requirements for large-scale structure surveys

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Imperfect photometric calibration of galaxy surveys due to either astrophysical or instrumental effects leads to biases in measuring galaxy clustering and the resulting cosmological parameter measurements. More interestingly (and disturbingly), the spatially varying calibration also generically leads to violations of statistical isotropy of the galaxy clustering signal. Here we develop, for the first time, a formalism to propagate the effects of photometric calibration variations with arbitrary spatial dependence across the sky to the observed power spectra and to the cosmological parameter constraints. We develop an end-to-end pipeline to study the effects of calibration, and illustrate our results using specific examples including Galactic dust extinction and survey-dependent magnitude limits as a function of zenith angle of the telescope. We establish requirements on the control of calibration so that it doesn't significantly bias constraints on dark energy and primordial non-Gaussianity. Two principal findings are: 1) largest-angle photometric calibration variations (dipole, quadrupole and a few more modes, though not the monopole) are the most damaging, and 2) calibration will need to be understood at the $\sim 0.1\%-1\%$ level (i.e. rms variations mapped out to accuracy between 0.001 and 0.01 mag), though the precise requirement strongly depends on the faint-end slope of the luminosity function and the redshift distribution of galaxies in the survey.

I. INTRODUCTION

Large-scale structure (LSS) measurements have become an extremely powerful probe of cosmology over the past 30 years. Starting with the pioneering Harvard-CfA survey [9], all the way to the Sloan Digital Sky Survey [34] and its extension Baryon Oscillation Sky Survey [8], Two-degree Field survey [5], and WiggleZ [11], the LSS surveys have revolutionized our understanding of the distribution of matter and energy in the cosmos, and helped impose percent-level constraints on the cosmological parameters (e.g. [3]).

A major challenge in current and future LSS surveys is understanding of the sample selection. We define calibration to be the measure of our understanding of the selection of our sample of galaxies, and calibration errors to be any unaccounted-for angular and redshift variations in the selection. The purpose of this paper is to determine how well calibration errors need to be controlled in order to avoid substantial degradation of the information we can extract from the LSS.

A particular source of uncertainty is known as *photometric calibration*. The term refers to the adjustments required to establish a consistent spatial and temporal measurement of flux of the target objects in the different bands of observation throughout the entire photometric survey. This is an enormous problem that all existing and upcoming wide area surveys face. The difficulty comes from the variability of various building blocks of the observational pipeline, which makes it difficult to establish a consistent flux baseline at each band (i.e. the flux zeropoints). In other words, because the instrument sensitivity is constantly changing, and so are the sources and intensity of noise, it is difficult to consistently compare

the fluxes for objects at different parts of the sky imaged at different times. Some examples of the manifestations of the photometric calibration errors in surveys are:

- *Detector sensitivity:* At any given time, the sensitivity of the pixels on the camera vary along the focal plane. In addition, the sensitivity of a given pixel can change with time.
- *Observing conditions:* Wide area surveys are carried out over several years and conditions are constantly changing. Observing conditions suffer from constant spatial and temporal variations.
- *Bright objects:* The light from foreground bright stars and galaxies affects the sky subtraction procedure, which impairs the surveys' completeness near bright objects [1, 28], and distorts the measured shapes of these faint galaxies [26].
- *Dust extinction:* Dust in the Milky Way absorbs light from the distant galaxies. As we show later, imperfect extinction correction can have serious consequences to cosmological clustering analyses.
- *Star-galaxy separation:* In photometric surveys, faint stars can be erroneously included in the galaxy sample. Conversely, galaxies are sometimes misclassified as stars and culled from the sample. These effects are important because stars are not randomly distributed across the sky.
- *Deblending:* Galaxy images can overlap, and it can be difficult to cleanly separate photometric and spectroscopic measurements for the blended objects.

Variabilities in the instrument sensitivity and observing conditions cause an angular variability in the depth of observations that the survey can achieve through each filter. Variations in the depth result in angular variations in the number density and redshift distribution of objects. In addition, because galaxy spectra are not flat, and because the sample selection involves more than one filter, depth variations cause variations in the angular and redshift distribution of galaxy types.

This variability in the sample selection can, in principle, be accounted for. This is not always done however, and it is common, for example, for correlation analyses of current data to assume a constant depth for the entire survey. Indeed, several sources of variability have been accounted for in the analysis of existing data – see in particular the pioneering work on the subject in the modern era of LSS surveys by Scranton et al. [31] (see also Vogeley [33]), and the more recent efforts by Ho et al. [17] and Ross et al. [28]. These authors modeled a wide variety of systematic errors, some of which qualify as the calibration errors (e.g. seeing, airmass, calibration offsets). In particular, the latter two papers identified bright stars as the major contaminant which adds significant power to the intrinsic clustering signal at large scales, and they applied two separate successful techniques to subtract this systematic contamination.

For the upcoming surveys an even more detailed analysis will be needed, ideally utilizing a formalism that is suited to a wide variety of photometric calibration systematics mentioned above and captures any kind of calibration-related systematic. One would also like to provide a guidance on how much calibration error, as a function of scale, can be tolerated in order not to degrade the cosmological parameter inferences. Here we aim to address both of these desiderata.

II. GOALS OF THIS WORK

In this paper we set out to study calibration errors in the most general way possible. Our goal is to build an end-to-end pipeline into which we can feed in calibration errors (or uncertainties) due to an arbitrary cause, and from which we obtain biases in cosmological parameters inferred from measurements of galaxy clustering in some LSS survey. We then turn the problem around, and estimate how well the calibration errors need to be controlled in order not to appreciably bias the cosmological parameter estimates.

To keep the scope of this paper reasonable, we only consider measurements of the galaxy two-point correlation function (i.e. the power spectrum), and leave other observable quantities – higher-order correlation functions of galaxies, for example – for future work. We also do not consider the effect of the photometric redshift errors which, while very important, are not expected to change our results in a major way, so we leave the photo-zs for a future analysis.

The paper is organized as follows. in Sec. III we describe our formalism of modeling both the true, underlying galaxy density field and the systematic errors describing variations in the photometric calibration. In Sec. IV we present the formalism to derive cosmological constraints and biases on cosmological parameters. In Sec. V we propagate the effects of the systematic errors to calculate the biases in the cosmological parameters. We conclude in Sec. VI. Important but technical details regarding various aspects of the computation of the effects of the photometric variation systematics on the observable quantities are relegated to the four Appendices.

III. FORMALISM: DESCRIBING SPATIALLY VARYING CALIBRATION

In this section we start by defining calibration errors and their field $c(\hat{\mathbf{n}})$, and proceed to derive the biased galaxy fluctuations in terms of this field in multipole space.

A. Calibration errors: definition and basics

Let true galaxy counts on the sky be denoted by $N(\hat{\mathbf{n}})$, where $\hat{\mathbf{n}}$ is an arbitrary spatial direction. The survey mean is given by $\bar{N} \equiv \langle N(\hat{\mathbf{n}}) \rangle_{\text{sky}}$, where the average here is taken over the observed sky. These true fluctuations in the galaxy counts can be expanded into harmonic coefficients $a_{\ell m}$ as

$$\frac{N(\hat{\mathbf{n}}) - \bar{N}}{\bar{N}} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}) \quad (1)$$

Consider a survey where a *deterministic* calibration error $c(\hat{\mathbf{n}})$ biases galaxy counts. In other words, given the true galaxy number counts in some direction $N(\hat{\mathbf{n}})$, the observed number is

$$N_{\text{obs}}(\hat{\mathbf{n}}) = [1 + c(\hat{\mathbf{n}})] N(\hat{\mathbf{n}}), \quad (2)$$

which implicitly defines the calibration field $c(\hat{\mathbf{n}})$. We can expand the calibration field relative to its fiducial value of zero (corresponding to no error)

$$c(\hat{\mathbf{n}}) = \sum_{\ell_1=0}^{\ell_{\text{calib,max}}} \sum_{m_1=-\ell_1}^{\ell_1} c_{\ell_1 m_1} Y_{\ell_1 m_1}(\hat{\mathbf{n}}), \quad (3)$$

where hereafter we assume that the calibration error dominates on large scales, and persists only out to some maximum multipole $\ell_{\text{calib,max}}$, corresponding to angular scale of $\pi/\ell_{\text{calib,max}}$ radians.

The statistical properties of the two galaxy number-density field, and the calibration-error field are, respectively

$$\langle a_{\ell m} \rangle = 0; \quad \langle a_{\ell m} a_{\ell' m'}^* \rangle = \delta_{mm'} \delta_{\ell\ell'} C_{\ell} \quad (4)$$

$$\langle c_{\ell m} \rangle = c_{\ell m}; \quad \langle c_{\ell m} c_{\ell' m'}^* \rangle = c_{\ell m} c_{\ell' m'}^* \quad (5)$$

Throughout the paper, angular brackets $\langle \cdot \rangle$ indicate ensemble averages, that is, averages over different realizations of the Universe. To reiterate, $N(\hat{\mathbf{n}})$ is the Gaussian random, isotropic field as predicted by inflation, while $c(\hat{\mathbf{n}})$ is a deterministic function given by calibration errors in the survey at hand.

In the remainder of this paper, we use the following definition: *Calibration variations (or errors) are departures of $c(\hat{\mathbf{n}})$, or its harmonic coefficients $c_{\ell m}$, from zero.* Our goal is to estimate how accurately those variations have to be known in order not to bias the cosmological parameter estimates.

Notice that we do not lose any generality by assuming that the c-field is fixed, rather than stochastic like the true galaxy density field. In this paper, we are effectively asking how much does this fixed systematic error bias the usual cosmological constraints. We are, of course, free to iterate over a number of specific incarnations of this 'fixed' error. A more specific example would be to ask how much does the Galactic dust pattern – its direction and amplitude fixed for the moment – bias some cosmological inference if unaccounted for perfectly, and then to repeat the analysis for a number of dust pattern realizations, or even different dust models.

B. Galaxy clustering and calibration errors: general case

In this Section we present the main results regarding the effect of the calibration errors on the observed clustering of galaxies and leave the details of the derivation to Appendix A.

The observed (i.e. biased) mean and density fluctuation are given by (see Eq. (A6)):

$$\begin{aligned} \delta^{\text{obs}}(\hat{\mathbf{n}}) &= \frac{1}{1+\epsilon} [\delta(\hat{\mathbf{n}})(1+c(\hat{\mathbf{n}})) + c(\hat{\mathbf{n}}) - \epsilon] \\ \bar{N}_{\text{obs}} &= \bar{N}(1+\epsilon) \end{aligned} \quad (6)$$

where

$$\epsilon \equiv \frac{1}{\bar{N}} \langle c(\hat{\mathbf{n}}) N(\hat{\mathbf{n}}) \rangle_{\text{sky}}. \quad (7)$$

The observed galaxy overdensity field can also be expanded in terms of the harmonic basis

$$\delta^{\text{obs}}(\hat{\mathbf{n}}) \equiv t(\hat{\mathbf{n}}) = \sum_{\ell m} t_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}) \quad (8)$$

The harmonic coefficients of δ^{obs} are calculated to be (see again Appendix A)

$$t_{\ell m} = \frac{1}{1+\epsilon} \left[a_{\ell m} + c_{\ell m} + \sum_{\ell_1 \ell_2 m_1 m_2} R_{m_1 m_2 m}^{\ell_1 \ell_2 \ell} c_{\ell_1 m_1} a_{\ell_2 m_2} \right] \quad (9)$$

where we dropped the $\ell = 0, m = 0$ term (see Eq. (A10) for the full expression) since we can not use the galaxy-fluctuation monopole to determine the cosmological parameters. The coupling matrix R can be written in terms of Wigner 3j symbols

$$\begin{aligned} R_{m_1 m_2 m}^{\ell_1 \ell_2 \ell} &\equiv (-1)^m \sqrt{\frac{(2\ell_1 + 1)(2\ell_2 + 1)(2\ell + 1)}{4\pi}} \\ &\times \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ m_1 & m_2 & -m \end{pmatrix}. \end{aligned} \quad (10)$$

The ensemble average of the multipole moments becomes, after some algebra

$$\langle t_{\ell m} t_{\ell' m'}^* \rangle = \frac{1}{(1+\epsilon)^2} \left\{ \underbrace{\delta_{mm'} \delta_{\ell\ell'} C_\ell}_{\text{isotropic}} + \underbrace{\left[U_{m'm}^{\ell\ell'} C_{\ell'} + (U_{mm'}^{\ell\ell'})^* C_\ell \right] + \sum_{\ell_2 m_2} U_{m_2 m}^{\ell_2 \ell} (U_{m_2 m'}^{\ell_2 \ell'})^* C_{\ell_2} + c_{\ell m} c_{\ell' m'}^*}_{\text{breaks statistical isotropy}} \right\}. \quad (11)$$

where we defined

$$U_{m_2 m}^{\ell_2 \ell} \equiv \sum_{\ell_1 m_1} c_{\ell_1 m_1} R_{m_1 m_2 m}^{\ell_1 \ell_2 \ell} \quad (12)$$

which is a function that depends on the Wigner 3j symbols as well as the calibration-field coefficients $c_{\ell m}$.

Equation (11) is the key result in this paper. As the label in the equation shows, the observed galaxy density field $t(\hat{\mathbf{n}})$ exhibits *broken statistical isotropy*. In particular, the variance of t is not rotationally invariant any more (i.e. it depends on m), and covariance between the

different ℓ modes is not zero any more. We can now utilize this formula and consider the isotropically measured power (i.e. assuming $\ell = \ell'$ and averaging over $m = m'$) and estimate how accurately any given systematic, described by the full set of $c_{\ell m}$, needs to be understood in order not to degrade the accuracy in measuring the cosmological parameters including non-Gaussianity¹.

C. Galaxy clustering and calibration errors: isotropic power case

We usually – essentially always, in fact! – assume that the field is isotropic, and then we use the data to calculate the correlation function, power spectrum, etc. Let us see how the assumed-isotropic angular power spectrum

is biased in terms of an arbitrary contamination field.

Setting $\ell = \ell'$ and $m = m'$ in Eq. (11), we get

$$\begin{aligned} \langle |t_{\ell m}|^2 \rangle &= \frac{1}{(1+\epsilon)^2} \left(\delta_{mm'} \delta_{\ell\ell'} C_\ell + 2 (U_{mm}^{\ell\ell})^{\text{Re}} C_\ell + \right. \\ &\quad \left. + \sum_{\ell_2 m_2} |U_{m_2 m}^{\ell_2 \ell}|^2 C_{\ell_2} + |c_{\ell m}|^2 \right) \end{aligned} \quad (13)$$

To assume statistical isotropy, we not only set $\ell = \ell'$ and $m = m'$ but further average over the $2\ell + 1$ values of m for a fixed ℓ . Then we obtain the prediction for the angular power spectrum that one would measure *assuming* statistical isotropy even when the systematics break it:

$$T_\ell \equiv \frac{\sum_{m=-\ell}^{\ell} \langle |t_{\ell m}|^2 \rangle}{2\ell + 1} = \frac{1}{(1+\epsilon)^2} \left[\left(1 + 2 \frac{c_{00}}{\sqrt{4\pi}} \right) C_\ell + \frac{\sum_{m=-\ell}^{\ell} (\sum_{\ell_2 m_2} |U_{m_2 m}^{\ell_2 \ell}|^2 C_{\ell_2} + |c_{\ell m}|^2)}{2\ell + 1} \right] \quad (14)$$

where $|U|^2 \equiv UU^*$, and where the sum over ℓ_2 goes in principle over all multipoles (though only those from the range $[\ell - \ell_{\text{calib},\text{max}}, \ell + \ell_{\text{calib},\text{max}}]$ are nonzero), while m_2 goes from $-\ell_2$ to ℓ_2 . Note that the term linear in U seen in Eq. (13) dramatically simplified in the expression for T_ℓ (Eq. 14) after we used the summation relation

$$\sum_{m=-\ell}^{\ell} (-1)^m \begin{pmatrix} \ell_1 & \ell & \ell \\ m_1 & m & -m \end{pmatrix} = (-1)^\ell \sqrt{2\ell + 1} \delta_{\ell_1 0}. \quad (15)$$

For a pure monopole calibration error (i.e. a pure c_{00} term), one can verify that the effects of the ϵ term and the c_{00} term in Eq. (14) exactly cancel and T_ℓ is unchanged. This makes intuitive sense, as a shift in the monopole changes the mean counts on the sky but does not affect the density *fluctuations*.

One can intuitively understand terms in Eq. (14):

- The $(1 + \epsilon)^{-2}$ prefactor accounts for the change in the mean number of galaxies observed on the sky; see Eq. (A7). As already mentioned, this term effectively ensures that the effect of the monopole (c_{00} , or constant change in calibration across the sky) is precisely canceled out.
- The terms containing $U_{m_2 m}^{\ell_2 \ell}$ introduce coupling between the multipoles. In particular, if there are calibration error multipoles out to some multipole

$\ell_{\text{calib},\text{max}}$, then there will be contributions in the range $[\ell - \ell_{\text{calib},\text{max}}, \ell + \ell_{\text{calib},\text{max}}]$ at a given ℓ , a fact familiar from spin coupling in quantum mechanics.

- Presence of the term $|c_{\ell m}|^2$ essentially means that the power of the calibration field is added to the intrinsic galaxy power spectrum. In other words, even if the distribution of galaxies on the sky were perfectly uniform so that $\delta \equiv N/\bar{N} - 1 = 0$ over some area, the calibration field will induce power so that $\delta^{\text{obs}} \neq 0$.

From the structure of Eq. (14), it is clear that calculating and storing the coefficients U is challenging. Naively, the problem requires calculation of the order of 10^{18} coefficients. Appendix D describes our approach of limiting the number of evaluations of $(\ell, \ell_1, \ell_2, m, m_1, m_2)$ and tabulating the coefficients U so that the number of operations is only of order 10^8 (for $\ell_{\text{max}} = 1000$ binned in ~ 30 multipole bins and considering the calibration variations out to $\ell_{\text{calib},\text{max}} = 20$), and is thus feasible.

We plot the biased power spectra T_ℓ further below in the next section.

Finally, it is worth writing down the observed angular *cross-correlation* power spectrum between fluctuations $t_{\ell m}^{(i)}$ and $t_{\ell m}^{*(j)}$ in two different tomographic redshift bins i and j ; it follows straightforwardly from Eq. (14) that:

$$T_\ell^{(ij)} = \frac{1}{(1 + \epsilon^{(i)})(1 + \epsilon^{(j)})} \left[\left(1 + \frac{c_{00}^{(i)} + c_{00}^{(j)}}{\sqrt{4\pi}} \right) C_\ell^{(ij)} + \frac{\sum_m \left(\sum_{\ell_2 m_2} (U_{m_2 m}^{\ell_2 \ell})^{(i)} (U_{m_2 m}^{\ell_2 \ell})^{*(j)} C_{\ell_2}^{(ij)} + c_{\ell m}^{(i)} c_{\ell m}^{*(j)} \right)}{2\ell + 1} \right] \quad (16)$$

where $c_{\ell m}^{(i)}$, $\epsilon^{(i)}$ and $(U_{m_2 m}^{\ell_2 \ell})^{(i)}$ are all evaluated in the redshift bin i (and same for j), and where $C_\ell^{(ij)}$ are the cross-correlation power spectra. While physical sources of calibration error are typically local and thus redshift-independent, in Sec. V we demonstrate that converting from the magnitude error to the calibration field $c(\hat{\mathbf{n}}) \equiv (\delta N/N)(\hat{\mathbf{n}})$ depends on the faint-end slope of the luminosity function, which typically *is* redshift-dependent, hence making the harmonic coefficients of $c(\hat{\mathbf{n}})$ also z -dependent and thereby potentially introducing couplings between the different redshift bins.

D. Additive and multiplicative systematics

Before we obtain numerical results on how some realistic calibration errors affect the observed power spectra, it pays to consider qualitatively how the power spectrum of galaxies is affected.

It is often useful to divide the effect of systematic errors into *additive* (those whose field is added to the true field observed on the sky), and *multiplicative* (those whose field multiplies the true field); see e.g. [13] and [15, 22] for the CMB and weak lensing systematic examples, respectively, where this nomenclature has been previously employed. These terms refer to the systematic error that either adds to the true galaxy fluctuations, or else multiplies it and modulates the true signal; see Eq. (6) for the real-space and Eq. (9) for the harmonic-space picture. For example, the angular parentheses on the right-hand side of Eq. (9) contain three terms; the term $a_{\ell m}$ corresponds to the true density field, $c_{\ell m}$ represents the additive effect of the systematic error, while the term containing $c_{\ell m} a_{\ell m}$ term together with the geometric factor R and the appropriate sum, represents the multiplicative effect of the systematics².

Additive and multiplicative errors in the counts translate into additive and multiplicative contributions from the calibration field to the observed galaxy power spectrum, see Eq. (11): the additive error is the term $c_{\ell m} c_{\ell m}^*$ while the multiplicative error are all terms involving the coupling matrix U . The two kinds of errors produce qual-

itatively different effects: additive error at some multipole ℓ_1 only affects power at that multipole, while the multiplicative error affects power at a range of multipoles; in particular, true power at an arbitrary multipole ℓ would leak to all multipoles in the range $[\ell - \ell_1, \ell + \ell_1]$.

The additive terms dominate the error budget on the largest scales, but are subdominant at smaller scales [33]. This can be understood qualitatively as follows: modulo geometric coupling terms, both additive and multiplicative terms are proportional to the square of coefficients $c_{\ell m}$, but the multiplicative terms are further multiplied by the fiducial angular power C_ℓ . Given that $C_\ell \ll 1$ at all ℓ and any redshift³, the multiplicative terms are suppressed relative to the additive terms. At higher multipoles, on the other hand, there are more ways in which power from other scales can leak into that ℓ so that the sums associated with multiplicative terms make them the dominant systematic contribution.

While it is often assumed for simplicity that the systematic errors, calibration or other, are represented by purely additive errors, we just demonstrated that both kinds of errors are important. In fact, in the plausible scenario where largest-scale information in the survey is ignored to avoid the systematic contamination, the multiplicative errors dominate. In what follows we use the full expressions containing both additive and multiplicative terms.

IV. POWER SPECTRA AND THEIR BIASES

In this section we propagate the effect of calibration errors to estimate biases in the cosmological parameters describing dark energy and primordial non-Gaussianity.

A. Fiducial cosmological model

We consider a set of cosmological parameters with the following fiducial values: matter density relative to critical $\Omega_M = 0.25$, equation of state parameter today $w_0 =$

² Technically speaking, the systematic effects are all multiplicative in Eq. (9) because of the $1/(1 + \epsilon)$ prefactor; however given that this prefactor is typically close to unity, the second and third term in parentheses of this equation act approximately in additive and multiplicative sense.

³ One exception – C_ℓ of order unity or larger – is realized in the scenario of galaxies at a very low redshift, $z \lesssim 0.05$. However such a sample would probably not be useful for cosmology given the significant galaxy peculiar velocities and the necessarily small volume probed. Moreover, the calibration errors would have less impact to begin with, since they would be affecting a very large intrinsic clustering signal.

-1 , its variation with scale factor $w_a = 0$, spectral index $n = 0.96$, and amplitude of the matter power spectrum $\ln A$ where $A = 2.3 \times 10^{-9}$ (corresponding to $\sigma_8 = 0.80$) defined at scale $k = 0.002 \text{Mpc}^{-1}$. Note that we hold fixed the Hubble constant $h = H_0/(100 \text{km/s/Mpc})$ (or equivalently, physical matter density $\Omega_M h^2$), the physical baryon density $\Omega_B h^2$, and we assumed a flat universe. On the other hand, we do not assume any other prior information, such as the CMB information from WMAP and Planck. In practice, this prior information would largely serve to fix the aforementioned three parameters. Note that this rather restricted set of assumptions about the set of cosmological parameters and external information about them is sufficient for our analysis: we are primarily concerned about the effect of the calibration systematics on the measured power spectra, and on the biases in the dark energy and non-Gaussianity parameters. Given that the systematics strongly depend on the properties of the galaxy sample and the survey (as we discuss further below), it is not necessary to model the up-to-date knowledge about the cosmological parameters in great detail.

We assume a survey covering 5,000 square degrees (so $f_{\text{sky}} \simeq 0.12$) with information out to $z_{\text{max}} = 1$, corresponding roughly to the Dark Energy Survey (DES). We assume that the number density of the galaxies is $n(z) \propto z^2 \exp(-z/z_0)$ with $z_0 = 0.3$, and that photometric redshifts enable splitting the sample in five tomographic bins centered at $z = 0.1, 0.3, 0.5, 0.7$ and 0.9 ; see the left panel of Fig. 1. The fiducial statistical constraints on the dark energy parameters are $\sigma(w_0) = 0.06$ and $\sigma(w_a) = 0.24$.

Instead of the power spectrum in wavenumber $P(k)$, we consider measurements of the angular power spectrum of galaxy fluctuations. In the Limber approximation, which is valid on intermediate to small angular scales, the angular power is given as (e.g. [21, 35])

$$\begin{aligned} P_\ell^{(ij)} &\equiv \langle a_{\ell m}^{(i)} a_{\ell m}^{*(j)} \rangle \\ &= \frac{2\pi^2}{\ell^3} \int_0^\infty r(z) H(z) b^2(k, z) \Delta^2(k, z) W_i(z) W_j(z) dz \end{aligned} \quad (17)$$

where i and j are referring to one of the five redshift bins⁴. The full, beyond-Limber expression, as well as the

definition of the $W(z)$ terms, are given in Appendix B. The power spectrum $\Delta^2(k, z) \equiv k^3 P(k)/(2\pi^2)$ is calculated using the transfer function output by CAMB, and its nonlinearities are modeled with the Smith et al. [32] formulae that were based on a halo model and fit to simulations. Appendix B has all of the details of how we calculate the power spectrum.

We consider information from multipoles $1 \leq \ell \leq 1000$, corresponding to spatial scales from about 10 arcmin to 180 degrees. To obtain accurate constraints on the parameter f_{NL} which come from large angular scales, we use every individual multipole between $\ell = 1$ and $\ell_{\text{calib},\text{max}} = 20$; beyond this we use ten more widely separated bins with $\Delta\ell \simeq 100$. Therefore, we use a total of 30 bins in ℓ ; for the low- ℓ ones we do *not* assume the Limber approximation and use Eqs. (B4), while for the higher multipoles we use the Limber approximation, Eq. (17) above.

Finally, we also allow for the presence of primordial non-Gaussianity. We adopt the widely studied ‘local’ model of non-Gaussianity

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{\text{NL}}(\Phi_G^2 - \langle \Phi_G^2 \rangle), \quad (18)$$

(where Φ is the primordial Newtonian gravitational potential, Φ_G is its Gaussian component, and f_{NL} is a dimensionless parameter), the bias becomes scale dependent, with a new term that goes as k^{-2} [7]

$$b(k) = b_0 + f_{\text{NL}}(b_0 - 1)\delta_c \frac{3\Omega_M H_0^2}{a g(a) T(k) c^2 k^2}, \quad (19)$$

where b_0 is the usual Gaussian bias (on large scales, where it is constant), $\delta_c \approx 1.686$ is the collapse threshold, a is the scale factor, Ω_M is the matter density relative to critical, H_0 is the Hubble constant, k is the wavenumber, $T(k)$ is the transfer function, $g(a)$ is the growth suppression factor, and c is the speed of light⁵. We assume the fiducial model with the Gaussian bias $b_0 = 2$ and zero non-Gaussianity, $f_{\text{NL}} = 0$.

The full set of cosmological parameters that we use is therefore

$$p_a \in \{\Omega_M, w_0, w_a, n, A, f_{\text{NL}}\}. \quad (20)$$

Given a systematic error in the power-spectrum, δC_ℓ , the resulting bias in the cosmological parameters, δp_a can be estimated using the Fisher matrix formalism as follows:

$$\delta p_a = \sum_j F_{ab}^{-1} \sum_{\ell,\alpha,\beta} \delta C_\ell^{(\alpha)} \text{Cov}^{-1} \left[C_\ell^{(\alpha)}, C_\ell^{(\beta)} \right] \frac{\partial C_\ell^{(\beta)}}{\partial p_j}, \quad (21)$$

⁴ While we use the all of the cross-correlations, $i \gtrsim j$, we notice that the cosmological constraints – and requirements on the systematics – remain unchanged if we only use the angular power spectrum auto-correlations with $i = j$. This is not too surprising, as auto-correlations $C_\ell^{(ii)}$ are guaranteed to be zero in the Limber approximation and in the absence of photometric redshift errors (photo-zs generically lead to overlap of redshift bins and thus bin-to-bin correlations). At large scales, where we do not employ the Limber approximation where it is not accurate (especially for nonzero f_{NL} which introduces very large-scale correlations), we explicitly verify that cross-powers do not appreciably alter our requirements on the control of the calibration errors.

⁵ This formula has higher-order corrections calibrated to N-body simulations and also derivable from theory, but these are typically small and not crucial for the present analysis so we ignore them here.

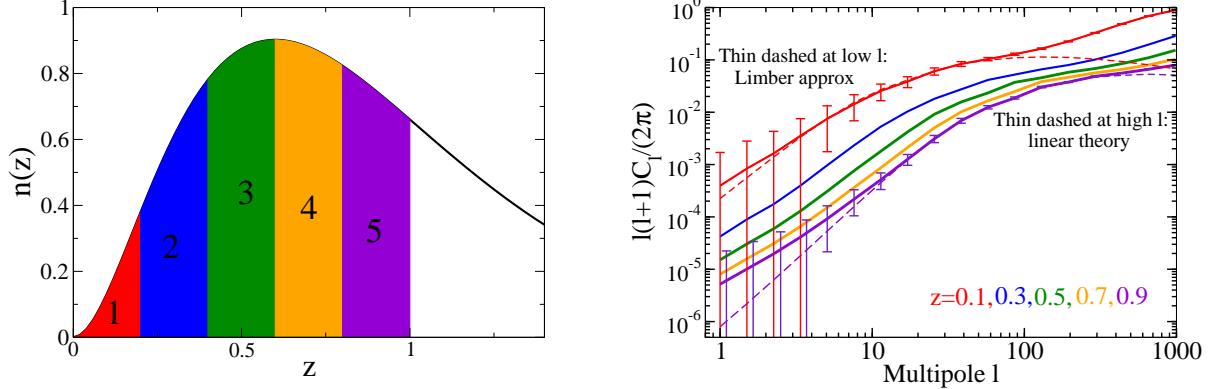


FIG. 1: Left panel: density distribution of galaxies assumed in this paper, and boundaries of the five redshift bins. We ignore information at $z > 1$, thus roughly modeling the difficulties with establishing accurate photometric redshifts at that range (for the DES). Right panel: Angular power spectra $C_\ell^{(ii)}$ for five redshift bins [Cross-correlations between the bins, while used in the analysis, are very small and not important nor shown in the figure.]. For the first and fifth bin we show, at the low multipole end, the full expression that we use at large scales (see Eq. (B4)) and, at the high-multipole end, the linear power spectrum for reference. For the first and fifth redshift bin we also show the cosmic variance errors plus shot noise.

where α and β stand for all pairs of bin indices (i, j) with $i \leq j$ (since $C_\ell^{(ji)} \equiv C_\ell^{(ij)}$) and the observed power spectrum is equal to the raw power plus shot noise

$$C_\ell^{(ij)} = P_\ell^{(ij)} + \delta_{ij} \frac{1}{N_i^{\text{sr}}} \quad (22)$$

where N_i^{sr} is the number of galaxies per steradian in the tomographic bin i . Moreover, δC_ℓ is calculated below as the difference between the observed, biased power spectrum T_ℓ and the fiducial C_ℓ . Assuming observations in the linear (and therefore Gaussian) regime only, the covariance matrix is the standard expression that follows using Wick's theorem:

$$\begin{aligned} \text{Cov} \left[C_\ell^{(ij)}, C_{\ell'}^{(kl)} \right] &= \frac{\delta_{\ell\ell'}}{(2\ell+1) f_{\text{sky}} \Delta \ell} \\ &\times \left[C_\ell^{(ik)} C_{\ell'}^{(jl)} + C_\ell^{(il)} C_{\ell'}^{(jk)} \right]. \end{aligned} \quad (23)$$

Using these equations, one can easily calculate the biases in the cosmological parameters for any given bias in the observed angular power spectra.

B. Biases in the observed power spectra

We would like to fairly compare the biases as a function of ℓ_1 , so we choose to adopt coefficients describing the uncertainties in the calibration field $c_{\ell_1 m_1}$ that lead to a *fixed variance* in the calibration pattern on the sky $c(\hat{\mathbf{n}})$ (and corresponding to, as we will shortly see, fixed variance in the angular variations of the magnitude limits

of the survey). Thus, we have

$$\text{Var}(c(\hat{\mathbf{n}})) = \frac{2\ell_1 + 1}{4\pi} C_{\ell_1}^{\text{sys}} = \frac{|c_{\ell_1 m_1}|^2}{4\pi} \quad (\text{no sum}) \quad (24)$$

since $C_{\ell_1}^{\text{sys}} = |c_{\ell_1 m_1}|^2 / (2\ell_1 + 1)$ is the angular power spectrum of the systematics (and really just the sum of their coefficients squared) and where the reader is reminded that, in this particular calculation we are “turning on” one (ℓ_1, m_1) pair at a time.

To consider the calibration variation in a single multipole (ℓ_1, m_1) that leads to a fixed variance in $c(\hat{\mathbf{n}})$, we therefore make the choice

$$c_{\ell_1 m_1}^{\text{Re,Im}} = \begin{cases} \sqrt{4\pi \text{Var}(c(\hat{\mathbf{n}}))} & (m = 0) \\ \sqrt{2\pi \text{Var}(c(\hat{\mathbf{n}}))} & (m \neq 0) \end{cases} \quad (25)$$

where in the $m \neq 0$ case both real and imaginary part of the $c_{\ell_1 m_1}$ have the given value.

Figure 2 shows the difference between the observed isotropic part of the power spectrum T_ℓ (see Eq. (14)) and the fiducial C_ℓ , divided by the statistical error (cosmic variance plus shot noise) for our assumed DES-type survey with $f_{\text{sky}} \simeq 1/8$. We assume a constant calibration error with rms⁶ of 0.01 or 0.001 per each multipole ℓ_1 separately. While a fixed ℓ_1 of the systematic errors affects all multipoles ℓ , it affects $\ell = \ell_1$ the most, so in this graph we only plot the effect on the observed power

⁶ For the purposes of making Figure 2 we put all of the photometric calibration variation in a single value of $m_1 = 0$.

spectrum at the *same multipole* at which the systematic errors occurs. In other words, Figure 2 shows the maximally affected multipole ℓ , for a fixed calibration variation error. The error in the measured tomographic angular power spectra decreases with multipole ℓ , which can be understood easily as follows: to a good approximation, the additional terms in the observed isotropic power spectrum, Eq. (14), are dominated by the additive term $|c_{\ell m}|^2/(2\ell+1)$. We find this is the case even for the lowest-redshift tomographic bin where C_ℓ is the highest and thus helps boost the multiplicative terms that contain the U coefficients. Moreover, generally we find that $\epsilon \simeq 0$. Therefore,

$$\frac{T_\ell - C_\ell}{\sigma(C_\ell)} \simeq \frac{\frac{|c_{\ell m}|^2}{(2\ell+1)}}{\sqrt{\frac{2}{(2\ell+1)f_{\text{sky}}}} \left(C_\ell + \frac{1}{N^{\text{sr}}} \right)} \quad (26)$$

where $C_\ell \equiv C_\ell^{(ii)}$ is referring to the power spectrum in some redshift bin i , and same for T_ℓ and $c_{\ell m}$. For a fixed variance in the photometric variations, $c_{\ell m}$ is independent of ℓ , so that this expression goes as $1/[\sqrt{2\ell+1}C_\ell]$, decreasing with ℓ very approximately as $\ell^{-1.3}$, at least out to $\ell = 10$ plotted in this Figure.

Figure 2 further shows that higher redshift bins are affected more than the lower redshifts. This is easy to understand: at higher redshift, the cosmological signal is smaller, given that it averages over more large-scale structure along the line of sight, and therefore it is more susceptible to the (redshift-independent) calibration bias.

The one important thing to take away from Fig. 2 is therefore that the calibration error is expected to be most damaging to the deepest surveys (or highest redshift slices of a survey). And it is precisely those highest redshifts that are most valuable in their information about dark energy and primordial non-Gaussianity.

V. SENSITIVITY TO CALIBRATION ERRORS

Let us now consider a few specific examples. First, we will study the sensitivities to an arbitrary systematic bias in calibration at each multipole *separately*, i.e. one (ℓ_1, m_1) pair at a time in the $c_{\ell_1 m_1}$. Then we study two concrete examples of effects that cause biases in calibration: corrections to the dust extinction maps, and variable survey depth.

We first illustrate the effect that a constant-magnitude calibration error with some spatial scale has on the biases in cosmological parameters. We then give two concrete examples of how the magnitude-limit variations can occur, and estimate their effect on the cosmological parameters using the formalism that we developed.

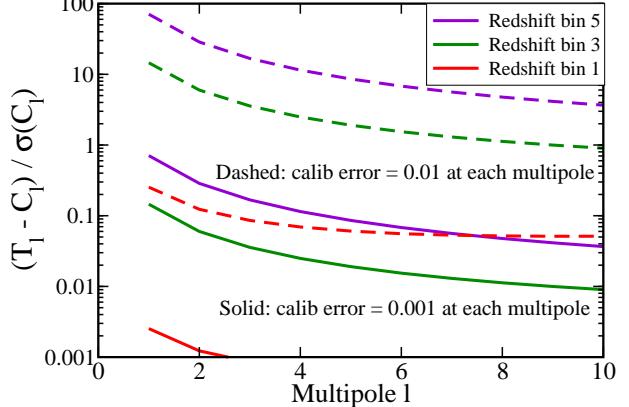


FIG. 2: Difference between the the observed isotropic part of the power spectrum T_ℓ (see Eq. (14)) and the fiducial C_ℓ , divided by the statistical error (cosmic variance plus shot noise) for our assumed DES-type survey with $f_{\text{sky}} \simeq 1/8$. We assume a constant rms photometric variation of 0.01 (dashed curves) or 0.001 (solid curves) per each multipole ℓ . Note that higher redshift bins are affected more than the lower redshifts. The fall-off with ℓ can be understood analytically; see text for details.

A. From calibration to galaxy counts

Consider the observed angular density of galaxies in some direction in the sky $N(\hat{\mathbf{n}}) \equiv \int n(z, \hat{\mathbf{n}}) dz$, where $n(z, \hat{\mathbf{n}})$ is the galaxy density in that direction and at redshift z .

Calibration errors correspond to variations in the magnitude limit of the survey $\delta m_{\text{max}}(\hat{\mathbf{n}})$. The observed density of galaxies changes since the galaxy density is a strong function of the survey depth (i.e. the magnitude limit). Suppressing the direction label $\hat{\mathbf{n}}$, we have

$$\begin{aligned} \delta[\log_{10} N(z, > m)] &= \frac{d \log_{10} N(z, > m)}{dm} \Big|_{m_{\text{max}}} \delta m_{\text{max}} \\ &\equiv s(z) \delta m_{\text{max}} \end{aligned} \quad (27)$$

where m_{max} is the maximal apparent magnitude observed in that direction in some waveband. It follows that the systematic bias in the observed fluctuations is

$$\left(\frac{\delta N}{N} \right)_{\text{sys}} = \ln(10) s(z) \delta m_{\text{max}}. \quad (28)$$

Often we have information about the selective extinction E_{B-V} ; the relation to magnitude extinction is $\delta m \equiv \delta A = \delta[R E_{B-V}]$ where R is the ratio of total to selective extinction and A is the alternative notation sometimes used for extinction. Assuming that R is known

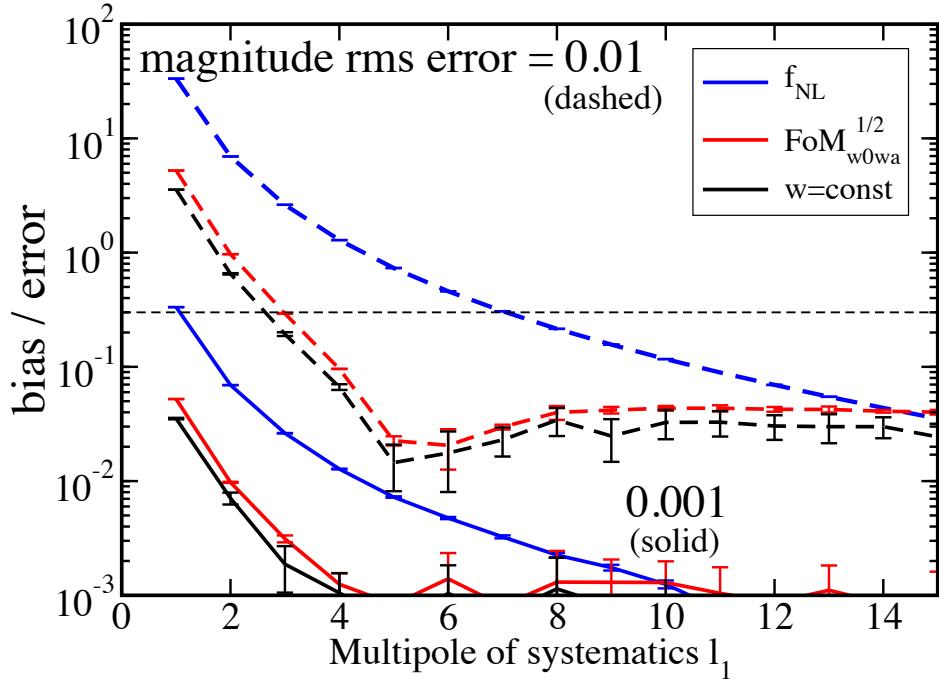


FIG. 3: Bias divided by marginalized statistical error in the cosmological parameters for the fixed magnitude root-mean-squared variation of 0.001 (solid curves) or 0.01 (dashed curves), as a function of multipole at which the systematics is introduced. To convert the magnitude variation to the $\delta N/N$ error, we used Eq. (28) and the best-fit faint-end slope of the luminosity function, $s(z)$, estimated from simulations of [23]; see text for details. The error bars show dependence on which of the m -values contains the calibration error; this dependence is small at the largest scales where the calibration error clearly has the largest effect. As discussed in the text, the monopole $\ell_1 = 0$ has no effect on the biases by definition. The line labeled $FOM^{1/2}$ refers to the square root of the DETF figure of merit, and serves to gauge any additional dependence brought forth by the temporal variation in the equation of state w_a . The dashed horizontal line denotes a fixed bias/error ratio of 0.3, which is approximately the upper limit of how much effect a systematic error should have on the cosmological parameters without seriously affecting the overall constraints in a survey.

perfectly⁷, $\delta m \simeq R \delta(E_{B-V})$, and thus (restoring the direction $\hat{\mathbf{n}}$ explicitly)

$$\left(\frac{\delta N}{N}\right)_{\text{sys}}(\hat{\mathbf{n}}) \equiv c(\hat{\mathbf{n}}) = \ln(10)s(z)\delta m_{\max}(\hat{\mathbf{n}}) \quad (29)$$

$$= \ln(10)s(z)R\delta(E_{B-V})(\hat{\mathbf{n}}). \quad (30)$$

While $s(z)$ is galaxy-population dependent, we can still estimate $(\delta N/N)_{\text{sys}}$ to be *very* roughly of order $\delta(E_{B-V})(\hat{\mathbf{n}})$, given that $s(z)$ is of order 0.1-1 while R takes values between about 1 and 5 depending on the band; see e.g. the appendix of Schlegel et al. [30] and important updates given in the Table 6 of Schlafly and Finkbeiner [29].

B. Calibration Bias per Multipole

Let us consider biases in our six cosmological parameters as a function of bias in a single multipole $c_{\ell_1 m_1}$. Following the prescription in the previous section, we assume that the variance of the calibration field is fixed and constant separately at each multipole ℓ_1 ; see Eq. (25).

We now propagate the calibration variation in a given (ℓ_1, m_1) , separately for $1 \leq \ell_1 \leq 20$ and $-\ell_1 \leq m_1 \leq \ell_1$, and for magnitude given in the above equations, to the observed angular power spectra via Eq. (14).

Figure 3 shows the bias divided by the statistical error in cosmological parameters for the fixed *magnitude* rms variation per multipole of $\langle \delta m_{\max}^2 \rangle_{\text{sky}}^{1/2} = 0.01$ or 0.001. We use Eq. (29) to translate this to the calibration-field variation, and then a modified version of Eq. (25) to calculate the harmonic coefficients $c_{\ell_1 m_1}$ that enter the calculation:

$$c_{\ell_1 m_1}^{\text{Re}, \text{Im}} = \begin{cases} \sqrt{4\pi \text{Var}(c(\hat{\mathbf{n}}))/(2\ell_1 + 1)} & (m = 0) \\ \sqrt{2\pi \text{Var}(c(\hat{\mathbf{n}}))/(2\ell_1 + 1)} & (m \neq 0) \end{cases} \quad (31)$$

⁷ If R is not perfectly known, as is often the case, then the magnitude variation is equal to the variation in the product between R and E_{B-V} .

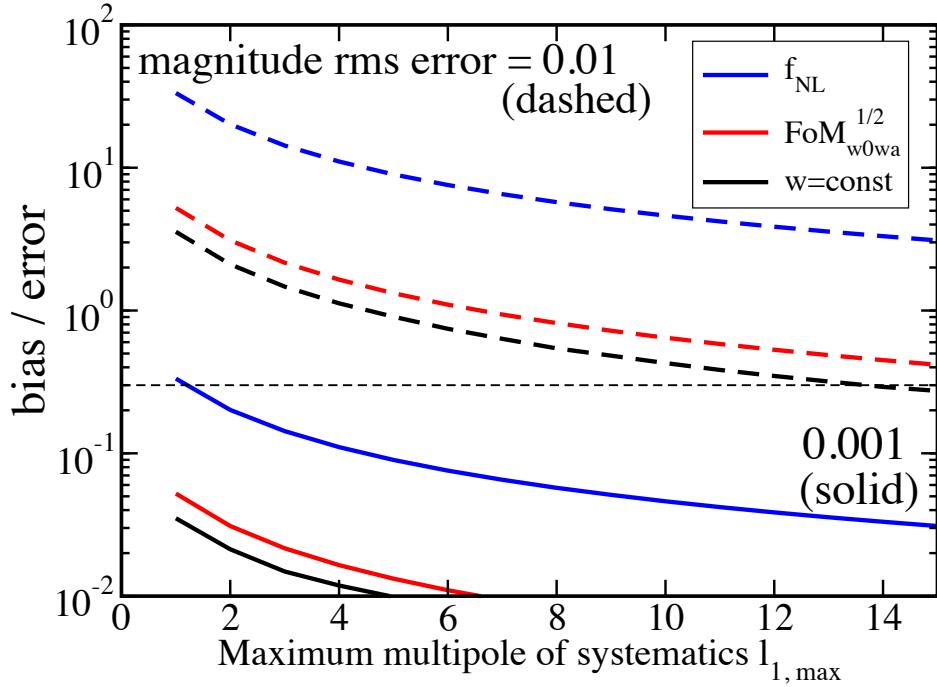


FIG. 4: Same as Fig. 3, except now the fixed magnitude error of 0.001 is *shared* equally among all multipoles in the range $1 \leq \ell_1 \leq \ell_{1,\max}$. Under this more realistic assumption than the fixed-multipole of error case shown in Fig. 3, the biases appear larger because the contributions from the largest scales dominate the budget at a fixed $\ell_{1,\max}$.

where, relative to Eq. (25), we have an additional term of $(2\ell_1 + 1)^{-1/2}$ to keep the variance in each ℓ_1 fixed since we are now distributing power over *all* m_1 -modes. We adopt the fiducial redshift-dependent faint-end slope of the luminosity function of

$$s(z) = 0.094 + 0.155z + 0.165z^2 \quad (32)$$

estimated from the simulations of [23], assuming a DES i-band magnitude limit of 24. This functional form roughly describes the trend that the higher redshifts are most affected by variations in the survey depth. We emphasize that this form for $s(z)$ is meant purely for illustration, as different galaxy samples will have different $s(z)$. We consider biases in the non-Gaussianity parameter f_{NL} , the (constant) equation of state of dark energy w , and the square root of the dark energy figure-of-merit, which is the inverse area of the 95% contour in the w_0-w_a plane [2, 20]. Note that the latter quantity takes into account the temporal variation of DE, and the square root serves to compare it fairly to the bias in constant w ; the two quantities, $\sigma(w)$ and $\text{FoM}^{1/2}$, show very similar behavior in these results. The error bar at each ℓ_1 shows the rms dispersion of the $(2\ell_1 + 1)$ values of m_1 into which we put the systematics. So, for example, at $\ell_1=6$ and for either one of the rms values for the calibration error, the error bars show the dispersion in the bias/error ratios for 13 different values of m_1 .

Figure 3 clearly indicates that systematic errors have the largest impact at largest angular scales assuming a

fixed contribution to the variance from each multipole. Bias in the non-Gaussianity parameter f_{NL} is larger than that for the dark energy parameters, which is expected because most of the information on f_{NL} comes from large angular scales which are particularly susceptible to calibration errors. For the calibration systematics at smaller scales – ℓ_1 beyond six or so corresponding to variation at scales less than about 30 degrees on the sky – the effect of the systematics asymptotes to a smaller value. Since calibration errors at large angular scales are the most damaging, it is sufficient to consider only those, and our choice of the maximum multipole of where the error enters, $\ell_1 \leq \ell_{\text{calib},\max} = 20$ is therefore sufficient.

Figure 4 is similar to Fig. 3 except now we show the effects when the magnitude error is split equally among all multipoles less or equal to $\ell_{1,\max}$ (instead of all of it being lumped in a single multipole as in Fig. 3). Given that the effect of the systematic error decreases with ℓ_1 , the biases in the cosmological parameters are larger than in the previous figure. Qualitatively, the two figures paint a consistent picture of the potentially deleterious effects of the calibration variations even at a level corresponding to $O(0.001\text{--}0.01)$ magnitudes.

C. Example I: Corrections to dust maps

We now study a specific scenario of calibration systematics: corrections to the SFD [30] dust extinction maps.

Dust in our Galaxy causes extinction, which in turn alters the observed galaxy fluctuations across the sky. While the Galactic dust has been accurately mapped out, we ask how accurately it *needs to* be mapped out in order not to bias the cosmological parameters.

To start, we need a model for the variations in the SFD map. We adopt results from the work of Peek and Graves [27] (hereafter PG10) who used ‘standard crayons’ – objects of known color – in the SDSS to correct the SFD maps over the north galactic cap region (for a related work, see Schlafly and Finkbeiner [29]). Schematically, therefore

$$\text{calibration variations} \equiv (\text{PG10} - \text{SFD}).$$

The SFD map is shown in the top left panel of Fig 5, while the PG10 *correction* is displayed in the top right panel. To convert this E_{B-V} map to $\delta N/N$ fluctuations (see Eq. (30)), we assume DES observations in the i-band, for which $R = 1.595$ [29].

The bottom left panel in Fig. 5 shows the angular power spectrum extracted using **Polspice** software package [4]. As explained in Appendix C at length, the most reliable way of modeling the calibration errors was to first extract the power from the map, then generate a full-sky realization *consistent* with that power (using the **isynfast** routine in HEALPix). We explicitly verified the intuitive expectation that the results do not depend much on the realization.

The bottom right of Fig. 5 shows the resulting biases in f_{NL} and the square root of the dark energy FoM, as a function of the faint-end slope of the luminosity function $s(z)$, assuming calibration variations are given by the PG10 corrections, and that $s(z)$ is constant in redshift. As mentioned around Eq. (28), the biases are very sensitive to the $s(z)$, scaling very nearly as $s(z)^2$. This can be easily understood: in Sec. III C we mentioned that the bias in the power spectrum is dominated by the added calibration power $\propto |c_{\ell m}|^2$, while the (real-space) calibration field is linear in $s(z)$ (Eq. 29); hence

$$\delta p_a \propto \delta C_\ell \propto |c_{\ell m}|^2 \propto s(z)^2. \quad (33)$$

Therefore, the faint-end slope of the luminosity function is a key factor relating the photometric magnitude variations to the cosmological parameter biases. Steep faint-end slopes will lead to particularly stringent requirements on our understanding of the large-angle photometric variations in the survey.

Regardless of the value of the faint-end slope, however, the bottom right panel of Fig. 5 shows that the effects of the imperfectly estimated Galactic dust on the cosmological parameters can be very significant.

D. Example II: variability of survey depth

Our second example is based on the expectations of photometric depth variations of the Dark Energy Survey.

We use a map (Jim Annis, private communication) simulating observations over 525 night of observation spread over 5-years; see the top panel of Fig. 6. The observing conditions on the site are based on historical atmospheric data of the CTIO site between 2005 and 2010. The survey tiling strategy uses multiple massive overlaps to generate as homogeneous as possible a survey. Each part of the sky is imaged ten times in each of the five DES filters (grizY). For simplicity, we only focus on the i-band survey-depth map.

The effect of the unaccounted-for variability in the survey depth is the same as that of the photometric calibration error. However, the variability is large and expected to be taken into account; therefore, we (arbitrarily) adopt the final calibration error to be equal to one-tenth of the depth-variation map (i.e. 1/10 of its amplitude shown in the top panel of Fig. 6). In other words, we assume

$$\text{calibration variations} \equiv \frac{1}{10} \times (\text{i-band variability map}).$$

We follow the same procedure as with the dust example above, and calculate the power spectrum of the depth variability map using **Polspice**; see the bottom left panel of Fig. 6. The variability of the survey depth will of course be accounted for in the data analysis – if it were not, it would lead to large biases in cosmological parameter estimates (as we easily verified using our formalism). The question is, then, to what accuracy do these variations need to be understood?

We answer that question by plotting, in the bottom right panel of Fig. 6, the bias in the (square root of the) DE FoM, and non-Gaussianity f_{NL} , as a function of the faint-end slope of the luminosity function $s(z)$. As in the previous example of the corrections to the SFD dust maps, we find that the biases in the cosmological parameters are significant, and that they strongly depend on the faint-end slope of the luminosity function. In fact, even assuming that only 10% of the variability in the survey depth is the ‘calibration error’ – the case shown in the bottom right panel of the Figure – the bias/error ratios are still large if $s(z) \gtrsim O(1)$.

VI. CONCLUSIONS

In this paper we made a first fully general study of the effect of the photometric calibration variations on the measured galaxy clustering angular power spectra. We derived a general formula for how a calibration variation with arbitrary spatial dependence affects the measured galaxy angular power spectrum. We illustrated the results assuming the standard set of cosmological parameters (including f_{NL}), DES-type dataset with five tomographic bins out to $z_{\max} = 1$, and two specific examples of real-world photometric redshift calibrations. We now summarize our findings.

Photometric variations modulate the observed angular distribution of galaxy counts according to Eq. (2).

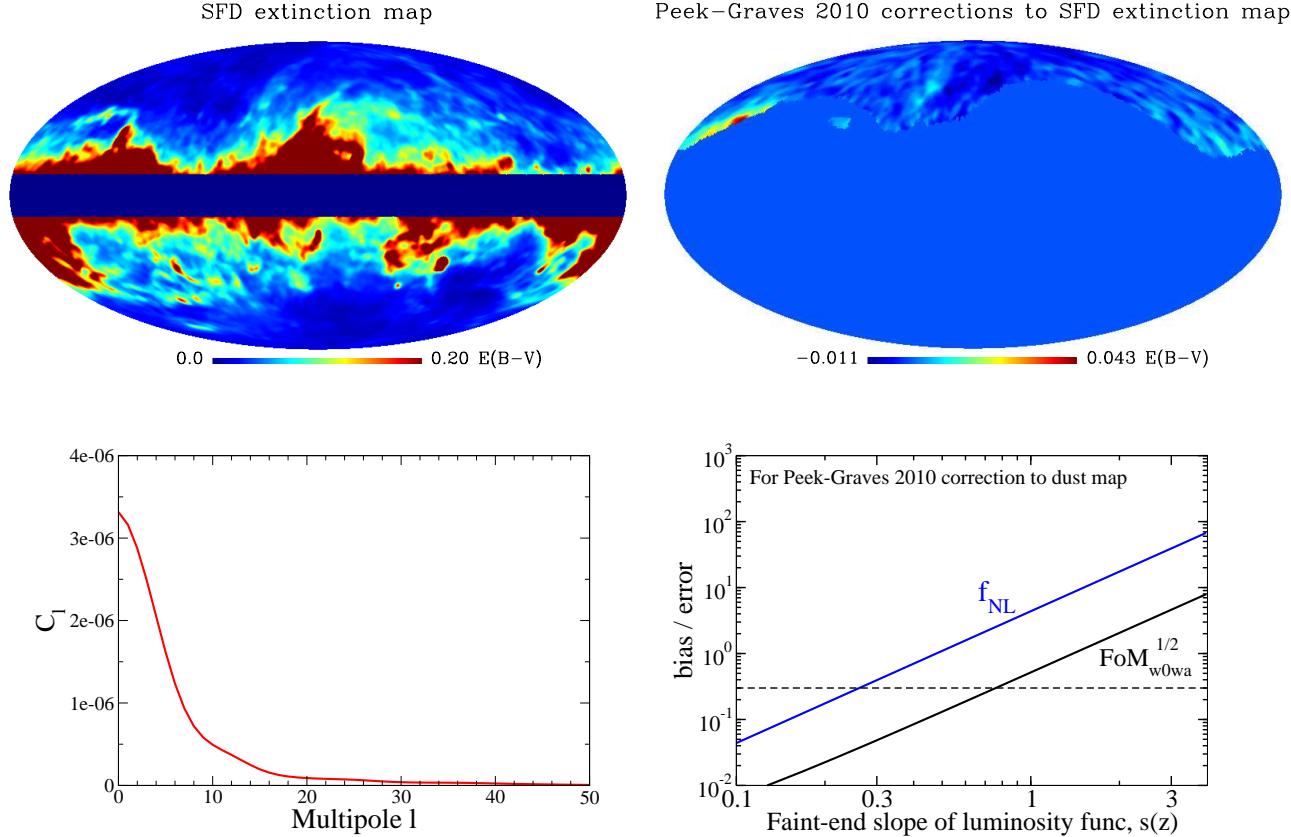


FIG. 5: Top left: Schlegel et al. [30] SFD extinction map, $E_{B-V}(\hat{\mathbf{n}})$, in Galactic coordinates with the 10 degrees Galactic plane cut. Top right: corrections to the SFD map from the work of Peek and Graves [27]. Bottom left: angular power spectrum of the PG10 map extracted by `Polspice` and shown without the usual $\ell(\ell+1)/(2\pi)$ term so that the relative contribution of different multipoles can be more easily seen. Bottom right: bias/error ratios for f_{NL} and the square root of the DETF FoM assuming PG10 map represents the calibration error, as a function of the faint-end slope of the luminosity function s . Note that the biases increase very sharply with s , roughly scaling as s^2 . The desired bias/error limit (horizontal dashed line) is exceeded already for $s \simeq 0.3$ for f_{NL} and $s \simeq 0.8$ for the dark energy equation of state.

This modulation translates into additive and multiplicative changes to the observed density fluctuation field, cf. Eqs. (6) and (9), which in turn generate additive and multiplicative changes to the observed power spectrum.

As shown in Eq. (11), photometric variations across the survey masquerade as apparent violations of statistical isotropy. Hence, explicit tests of statistical isotropy could provide a useful way to identify unaccounted-for variations in the photometry. In this paper, we focused on the effects in the angle-averaged power-spectrum, cf. Eq. (14). We found that large-angle modulations of power (dipole, quadrupole, etc), are particularly damaging to cosmological analysis. We demonstrate this explicitly (cf. Eq. (26) and Fig. 2) for the case where the variance in the photometric calibration error field is concentrated in one multipole ℓ_1 at a time. Note that the spatially uniform photometric decrement or increment across the sky (i.e. the monopole, $\ell_1 = 0$) is unobservable since it only affects the mean number of galaxies in the survey.

Specializing in the angle-averaged power spectrum as done in Eq. (14), one can explicitly show that largest-angle fluctuations are dominant (for a fixed induced variance on the calibration error field $c(\hat{\mathbf{n}})$). (Fig. 2). Moreover, highest-redshift clustering measurements are most susceptible to the photometric variations, essentially because their angular power is the smallest and thus is more affected by the photometric variation.

Less obviously, we find that the additive errors (e.g. term proportional to $|c_{\ell m}|^2$ in Eq. (14)) are typically dominant over the multiplicative biases (terms proportional to the coefficients U) for all redshift bins and at large angular scales. The reason is simple: because they couple different multipoles, multiplicative terms are suppressed relative to the additive ones by the fiducial angular power spectrum C_ℓ factor; see the term with C_{ℓ_2} in Eq. (14). Since $C_\ell \ll 1$ even at low-z (and all ℓ), the additive terms dominate the error budget if all ℓ modes are used in the analysis. However, at slightly smaller angular scales ($\ell \gtrsim 10$) the multiplicative error terms dominate

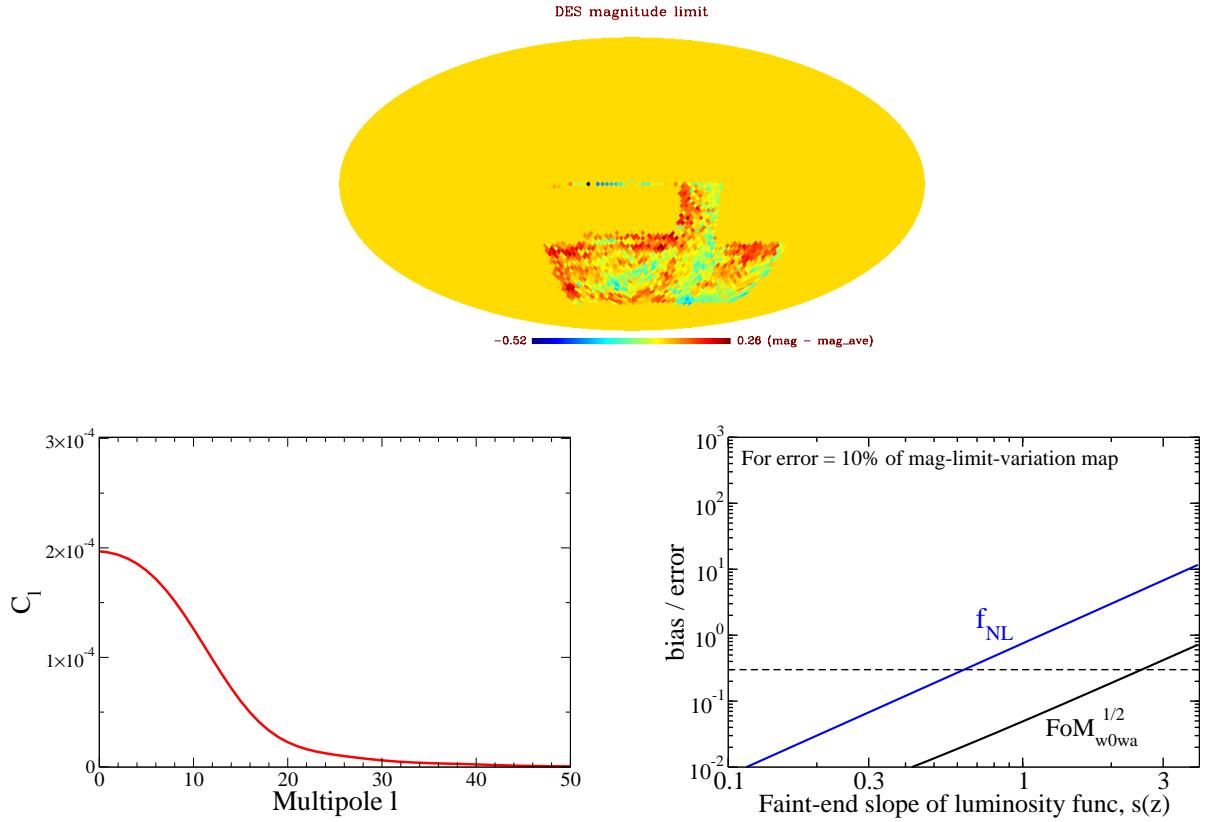


FIG. 6: Top panel: i-band magnitude limits estimated for the upcoming observations of the Dark Energy Camera at CTIO as a function of angular position. The pattern of variations in the magnitude limits are set by the variations in the observing conditions and the survey tiling strategy over the five years of the survey. Bottom left: power spectrum of the map on the left, extracted using `Polspice` and shown without the usual $\ell(\ell+1)/(2\pi)$ term so that the relative contribution of different multipoles can be more easily seen. Bottom right: biases in the cosmological parameters vs. the faint-end slope of the luminosity function $s(z)$ assuming calibration error maps is consistent with a fixed *fraction* of 10% of amplitude (or 1% of power) of the magnitude-limit map shown in the top (bottom left) panel. The desired bias/error limit (horizontal dashed line) is exceeded for $s(z) \simeq 1$.

the error budget and can significantly bias the cosmological constraints, as discussed in Sec. V. Therefore, it is important to include both multiplicative and additive aspects of the calibration error to accurately model biases in cosmological analyses.

The photometric variation calibration errors affect the cosmological parameter measurements at largest observable scales biasing dark energy and, especially, primordial non-Gaussianity, the latter of which is intrinsically has signal at large scales and is particularly susceptible to these systematic errors. Figures 3 and 4, the principal plots in this paper, show these cosmological parameter biases for a fixed contribution of the calibration error at each multipole separately and for a range of multipoles, respectively. In Sec. V we further give two specific real-world examples of what the photometric variations could occur: errors in mapping the dust in our Galaxy, and variations in survey depth. We find that these calibration errors lead to potentially large cosmological biases, especially if the faint-end slope of the luminosity function

$s = d \log N / dm|_{m_{\max}}$ is steep. In particular, in the Fisher matrix approximation, the cosmological biases scale as s^2 .

As a by-product of this work, we developed a reasonably fast algorithm, and provide a code, to calculate the full biased angular power spectrum of galaxies given an arbitrary photometric calibration variation map on the sky. As mentioned above, this calculation *would* be rather trivial if we could assume that the power spectrum errors are purely additive; however we demonstrated that multiplicative errors are important and may in fact dominate, necessitating the more numerically intensive calculation for surveys of interest.

From our analyses, it appears that the total rms of the calibration error has to be kept at the level at somewhere between 0.001 and 0.01 magnitudes rms, depending on exactly to how large a scale one wants to push in order to maximize extraction of cosmological information, in order not to bias the cosmological parameters appreciably. This is a very stringent requirement! Achieving

it, however, may not be as difficult as it sounds given that this is the time-averaged error to be tolerated at the end of the whole survey. Moreover, there are several other tools that we have at our disposal that we did not consider in this preliminary work. For example, one could use the survey itself to internally determine (“self-calibrate”) the photometric-variation errors, much like similar weak lensing, cluster count, or type Ia supernova surveys are doing or planning to do with their systematic errors. One could also use measurements of the higher-point correlation functions to help determine these nuisance parameters (in our language, the $c_{\ell_1 m_1}$). We leave these promising avenues for future study.

Acknowledgements

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Appendix A: Derivation of the calibration-induced bias on the measured angular power spectrum

Consider a survey where fixed, deterministic calibration error, $c(\hat{\mathbf{n}})$, biases galaxy counts. In other words, given the true galaxy number counts in some direction $N(\hat{\mathbf{n}})$, the observed number is

$$N_{\text{obs}}(\hat{\mathbf{n}}) = [1 + c(\hat{\mathbf{n}})] N(\hat{\mathbf{n}}). \quad (\text{A1})$$

Let us expand the various quantities in harmonic space. First of all, true (i.e. no-systematic-errors) fluctuations in the galaxy counts can be expanded in the spherical harmonic basis

$$\frac{N(\hat{\mathbf{n}}) - \bar{N}}{\bar{N}} = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}), \quad (\text{A2})$$

where, notice, the monopole is zero by definition of the left-hand side. Meanwhile, we can expand the calibration field relative to its fiducial value of zero (corresponding to no error)

$$c(\hat{\mathbf{n}}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} c_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}) \quad (\text{A3})$$

Notice the statistical properties of the two fields

$$\langle a_{\ell m} \rangle = 0; \quad \langle a_{\ell m} a_{\ell' m'}^* \rangle = \delta_{mm'} \delta_{\ell\ell'} C_\ell \quad (\text{A4})$$

$$\langle c_{\ell m} \rangle = c_{\ell m}; \quad \langle c_{\ell m} c_{\ell' m'}^* \rangle = c_{\ell m} c_{\ell' m'}^* \quad (\text{A5})$$

in other words, $N(\hat{\mathbf{n}})$ is the Gaussian random, isotropic field as predicted by inflation, while $c(\hat{\mathbf{n}})$ is a deterministic function given by calibration errors in a survey.

Let us now calculate the observed density contrast of galaxies:

$$\begin{aligned} \delta^{\text{obs}}(\hat{\mathbf{n}}) &\equiv \frac{N_{\text{obs}} - \bar{N}_{\text{obs}}}{\bar{N}_{\text{obs}}} \\ &= \left[\frac{N(1 + c(\hat{\mathbf{n}}))}{\bar{N}(1 + \epsilon)} - 1 \right] \\ &= \frac{1}{1 + \epsilon} [\delta(\hat{\mathbf{n}})(1 + c(\hat{\mathbf{n}})) + c(\hat{\mathbf{n}}) - \epsilon] \\ &= \frac{1}{1 + \epsilon} \left[\delta(\hat{\mathbf{n}}) \left(1 + \sum_{\ell, m} c_{\ell m} Y_{\ell m} \right) + \sum_{\ell, m} c_{\ell m} Y_{\ell m} - \epsilon \right] \end{aligned} \quad (\text{A6})$$

where $Y_{\ell m} \equiv Y_{\ell m}(\hat{\mathbf{n}})$, and we expanded the photometric calibration variation field $c(\hat{\mathbf{n}})$ into spherical harmonics. Here ϵ is the relative bias in the measured mean of galaxies:

$$\begin{aligned} \bar{N}_{\text{obs}} &\equiv \langle N_{\text{obs}}(\hat{\mathbf{n}}) \rangle_{\text{sky}} \\ &= \bar{N} + \sum_{\ell m} c_{\ell m} \langle N(\hat{\mathbf{n}}) Y_{\ell m}(\hat{\mathbf{n}}) \rangle_{\text{sky}} \\ &\equiv \bar{N}(1 + \epsilon) \end{aligned} \quad (\text{A7})$$

(the $\langle \rangle_{\text{sky}}$ denotes sky average), so that

$$\epsilon \equiv \frac{1}{\bar{N}} \sum_{\ell m} c_{\ell m} \langle N(\hat{\mathbf{n}}) Y_{\ell m}(\hat{\mathbf{n}}) \rangle_{\text{sky}}. \quad (\text{A8})$$

It is now rather straightforward to calculate the multipole coefficients of the observed density field. Writing

$$\delta^{\text{obs}}(\hat{\mathbf{n}}) = \sum t_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}), \quad (\text{A9})$$

equating to the last line of Eq. (A6) and inverting by multiplying with $Y_{\ell m}^*$ and using the orthogonality relation, we have

$$\begin{aligned} t_{\ell m} &= \frac{1}{1 + \epsilon} \left[a_{\ell m} + c_{\ell m} + \sum_{\ell_1 \ell_2 m_1 m_2} R_{m_1 m_2 m}^{\ell_1 \ell_2 \ell} c_{\ell_1 m_1} a_{\ell_2 m_2} \right. \\ &\quad \left. - \sqrt{4\pi\epsilon} \delta_{\ell 0} \delta_{m 0} \right] \end{aligned} \quad (\text{A10})$$

where to obtain the last term in the last line we used $1 = \sqrt{4\pi} Y_{00}$.

Taking the two-point correlation of $t_{\ell m}$ is now straightforward, and things are simplified because all terms proportional to a single power of $a_{\ell m}$ (or its conjugate) vanish – remember that $c_{\ell m}$ are just some numbers here. Moreover, we can ignore the term proportional to $\delta_{\ell 0}$ since it only affects the monopole which is not used in cosmological constraints. Thus we recover Eq. (11):

$$\langle t_{\ell m} t_{\ell' m'}^* \rangle = \frac{1}{(1+\epsilon)^2} \left\{ \delta_{mm'} \delta_{\ell\ell'} C_\ell + \left[U_{m'm}^{\ell\ell'} C_{\ell'} + (U_{mm'}^{\ell\ell'})^* C_\ell \right] + \sum_{\ell_2 m_2} U_{m_2 m}^{\ell_2 \ell} (U_{m_2 m'}^{\ell_2 \ell'})^* C_{\ell_2} + c_{\ell m} c_{\ell' m'}^* \right\}. \quad (\text{A11})$$

What about ϵ ? It can be evaluated directly in real space, given the calibration error map. Or it can be calculated in harmonic space, starting from its definition in Eq. (A8) combined with Eq. (1), and using the orthogonality theorem for spherical harmonics

$$\langle Y_{\ell m} Y_{\ell' m'} \rangle_{\text{sky}} = \frac{(-1)^m}{4\pi} \delta_{\ell\ell'} \delta_{m(-m')}. \quad (\text{A12})$$

The result is

$$\epsilon = \frac{c_{00}}{\sqrt{4\pi}} + \sum_{\ell,m} \frac{c_{\ell m} a_{\ell m}^*}{4\pi} \quad (\text{A13})$$

where we used the identity $(-1)^m a_{\ell(-m)} = a_{\ell m}^*$. In cases where $f_{\text{sky}} < 1$, the orthogonality relation in Eq. (A12) does not hold, but Eq. (A13) still does if the coefficients $a_{\ell m}$ are interpreted as the cut-sky harmonics of the density field.

Appendix B: Angular power spectra and their information content

We model the angular power spectra of galaxy density fluctuations as follows. In Limber's approximation, the angular power spectrum is given by

$$P_\ell^{(ij)} = \frac{2\pi^2}{\ell^3} \int_0^\infty dz r(z) H(z) W_i(z) W_j(z) \Delta^2 \left(k = \frac{\ell}{r(z)}, z \right), \quad (\text{B1})$$

where $\Delta^2(k) \equiv k^3 P(k)/(2\pi^2)$ is the dimensionless power spectrum, $r(z)$ is the comoving angular diameter distance and $H(z)$ is the Hubble parameter. The weights W_i are given by

$$W_i(z) = \frac{n(z)}{\int_{z_{\min}^{(i)}}^{z_{\max}^{(i)}} n(z) dz} [H(z_{\max}^{(i)}) - H(z_{\min}^{(i)})], \quad (\text{B2})$$

where $H(x)$ is the Heaviside step function and $z_{\min}^{(i)}$ and $z_{\max}^{(i)}$ are the lower and upper bound of the i th redshift bin and $n(z)$ is the normalized radial distribution of galaxies⁸

whose form we take to be

$$n(z) = \frac{1}{2z_0^3} z^2 e^{-z/z_0}, \quad (\text{B3})$$

which peaks at $z_{\max} = 2z_0$. In this work we use $z_0 = 0.3$ so that the radial distribution peaks at redshift 0.6.

At the lowest multipoles the Limber approximation is not accurate any more; see Fig. 1. At $\ell \leq 30$ we adopt the full expression for the power spectrum; using notation from e.g. [14] this is

$$\begin{aligned} P_\ell^{(ij)} &= 4\pi \int_0^\infty d \ln k \Delta^2(k, z=0) I_i(k) I_j(k) \\ I_i(k) &\equiv b_i(k) \int_0^\infty dz W_i(z) \frac{D(z)}{D(0)} j_\ell(k \chi(z)) \end{aligned} \quad (\text{B4})$$

where $\chi(r)$ is the radial distance, and $\chi(z) = r(z)$ in a flat universe that we consider. Here $D(z)$ is the linear growth function of density fluctuations, so that $\delta(z) = [D(z)/D(0)]\delta(0)$. At multipoles $\ell > 30$ we continue to use the Limber approximation since the computation is much faster. The power spectra are shown in Fig. 1. Note that, when using Eq. (B4), the non-linear corrections to clustering are negligible since we only apply this equation at very large scales.

Figure 7 shows the contribution to the signal-to-noise squared for each cosmological parameter separately per multipole – in other words, we show contribution to the Fisher matrix element F_{aa} for each parameter p_a from a single multipole ℓ . There is a clear trend of increased information at high multipoles, except for f_{NL} whose information largely comes at the lowest multipoles, despite cosmic variance [7].

Appendix C: Extraction of full-sky calibration from cut-sky data

In order to estimate the effect of photometric calibration errors, we perform all calculations in multipole space. In particular, our formalism requires the multipole coefficients of the calibration variation field, $c_{\ell_1 m_1}$, as input. In practice, this job will be left to observers who will have a number of tools at their disposal to estimate the photometric variation in their survey (in magnitudes, say, as a function of sky position) in their survey. They, or their theory friends, can then use our formalism, Eq. (14), to estimate the effect of calibration variations on the angular power spectra and the cosmological parameters.

⁸ Note that a sometimes-used alternative definition of $n(z)$ refers to the spatial density of galaxies (e.g. Hu and Jain [19]); it is related to the quantity we use via $dN/dz = n(z) \Omega r^2(z)/H(z)$, where Ω is the solid angle spanned by the survey, and r and H are the comoving distance and Hubble parameter, respectively. Note also that our $W(z)$ is equivalent to the quantity $f(z)$ from Ho et al. [16].

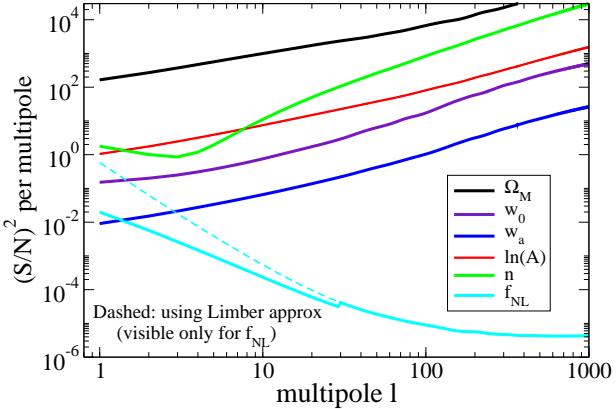


FIG. 7: Signal-to-noise squared contribution (i.e. contribution to diagonal Fisher matrix element with no systematics present) per single multipole ℓ for each cosmological parameter. Note that we use the Limber approximation only at $\ell > 30$ in order to more accurately capture information on f_{NL} at $\ell \leq 30$. The reason for the small discontinuity at $\ell = 30$ is due to the switch from full to Limber power spectrum.

However it is nontrivial to obtain the coefficients $c_{\ell_1 m_1}$ given the cut-sky observations, i.e. partial sky coverage: recall, for example, that the Peak-Graves dust-correction map from Sec. VC covers about 1/4 of the sky, while the DES depth map from Sec. VD covers only 1/8 of the sky. Note that the naive reconstruction from the observed map, $c_{\ell_1 m_1} = \int Y_{\ell_1 m_1}^*(\hat{\mathbf{n}}) \delta N / N(\hat{\mathbf{n}}) d\Omega$, would give the *cut-sky* multipoles which describe the field that is zero outside of the observed region, which we do *not* want: the cut-sky multipoles have wrong amplitudes and harmonic structure relative to the full-sky ‘truth’.

Filling in the missing sky is a well-known problem in the cosmic microwave background literature. While the reconstruction of the power C_ℓ is relatively straightforward, reconstruction of the temperature field $a_{\ell m}$ is challenging and typically works well only when substantial portions of the sky ($f_{sky} \gtrsim 0.8$) are observed. Fortunately, in this work we are only interested in obtaining *approximate* values for the $c_{\ell_1 m_1}$, consistent with the cut-sky map of the calibration field, in order to estimate its effects on cosmological parameters.

We have attempted, without success, two rather well-known techniques for reconstructing the true, full-sky $c_{\ell_1 m_1}$ from partial sky observations:

- We tried the direct reconstruction of the field, using the maximum-likelihood procedure (see e.g. [6, 10, 12]); this approach is used, for example, to reconstruct the CMB power at $\ell \lesssim 30$ [25]. While this approach returns the true structure in the observed area of the sky (as it must), the recon-

structions add spurious structure and extra power – clearly visible by eye – in the unobserved areas of the sky. This remains the case even after extensive testing and fine-tuning of the procedure, and it only returned trustworthy results at high sky coverage ($f_{sky} \gtrsim 0.8$). Therefore, this approach fails to extrapolate the photometric-variation field beyond the observed region even approximately.

- We tried “harmonic inpainting” [18, 24] of our two maps, were the missing portion of the sky is filled in assuming that the density field is Gaussian, and starting with a guess for the covariance matrix of the field (which is just given in terms of the theoretical angular power spectrum). As with the direct reconstruction, we find large spurious power in the unobserved portion of the sky in realistic cases when sky coverage is small, $f_{sky} \ll 1$. The negative results persist even after attempts to fine-tune the analysis to the maps at hand, again in parallel with the reconstruction method.

Given these failures and the fact that we only need a rough estimate of the full-sky calibration field $c_{\ell_1 m_1}$, we resort to the simpler scheme of reconstructing the large-angle power spectrum and drawing realizations of the field *consistent* with it. In particular, we

1. Calculate the power spectrum using **Polspice** package [4] to calculate the angular power spectrum from the cut-sky map. **Polspice** uses the pixel-based approach to calculate the real-space angular power first and then, with appropriate apodizations, convert it to the C_ℓ .
2. Sample from these C_ℓ , i.e. $c_{\ell_1 0} = \mathcal{N}(0, C_{\ell_1})$ and $c_{\ell_1 m_1}^{\text{Re},\text{Im}} = \mathcal{N}(0, C_{\ell_1}/2)$ for $m_1 \neq 0$, where $\mathcal{N}(\mu, \sigma^2)$ is the Gaussian normal function with mean μ and variance σ^2 .
3. Repeat the previous step a number of times to ensure appropriate averaging over realizations, though in practice we find that the each realization (for a fixed estimated C_{ℓ_1}) leads to similar results.

The approach returned a smooth power spectrum at large scales (see the C_ℓ panels in Figs. 5 and 6), and therefore reliable multipole coefficients describing calibration variations across the whole sky.

The reader might be worried that, in the above-mentioned successful approach, we assumed Gaussianity for the calibration field by assuming that the C_{ℓ_1} describes the full-sky field, and moreover assuming that the $c_{\ell_1 m_1}$ are Gaussian random variates. This is definitely an assumption needed to get the coefficients *in our particular examples* described in this Appendix but not in general. Moreover, we are not concerned about the assumption of Gaussianity of the calibration-variation field: Eq. (11), for example, illustrates that the calibration field will break statistical isotropy of the observed

density fluctuations regardless of the nature of the statistical distribution of the ensemble from which the $c_{\ell_1 m_1}$ come from. Modest dispersion among the different m_1 -modes in Fig. 3 confirms that the principal property of these coefficients that drives their effect on cosmology are their amplitudes as a function of ℓ_1 , not their phases.

We emphasize here that the somewhat-challenging task of needing to determine the (full-sky) $c_{\ell_1 m_1}$ discussed in this Appendix is largely orthogonal to the overall goal of this paper of quantifying the effects of photometric variations. In practice, the observers will have an opportunity to estimate the photometric calibration variations across the sky in real space using a number of methods (e.g. using theoretical estimates, or measurements across part of the sky as we did in our examples). Converting from $c(\hat{\mathbf{n}})$ to the full-sky $c_{\ell_1 m_1}$ is then just a purely mathematical exercise in which only an approximate answer is required.

Appendix D: Tabulation of coefficients

In order to speed up calculations of the observed galaxy overdensity field $t_{\ell m}$ and its correlations in harmonic space, we need to calculate a very large number of geometrical quantities $R_{m_1 m_2 m}^{\ell_1 \ell_2 \ell}$; see Eqs. (10)-(12). Recall that each (ℓ, m) pair has $2\ell+1$ quantities for a fixed m , or approximately ℓ_{\max}^2 quantities for all ℓ up to some ℓ_{\max} . Calculating all of the R coefficients would then naively involve $O(\ell_{\max}^6)$ operations which, for $\ell_{\max} \simeq 1000$, would be 10^{18} calculations of the Wigner-3j symbols! Calculating even a fraction of such a large number of coefficients is clearly unfeasible.

We bring the computation required to a manageable size and speed as follows. We decide to tabulate the coefficients U for the minimum required number of quadruplets of its indices, and carry out the summation in Eq. (12) for each U . Moreover, since our goal is to concentrate on large-angle (low- ℓ_1) systematics, which limits the maximum multipole of the *systematics*, ℓ_1 (see Eq. (12)), we choose $\ell_1 \leq \ell_{\text{calib,max}}$ with $\ell_{\text{calib,max}} = 20$. Therefore, the total number of evaluations of the Wigner-3j symbols will be approximately equal to the number of (ℓ_1, m_1) pairs times the number of the U coefficients; the former quantity is

$$N_{\ell_1, m_1} = \sum_{\ell_1=0}^{\ell_{\text{calib,max}}} (2\ell_1 + 1) = (\ell_{\text{calib,max}} + 1)^2, \quad (\text{D1})$$

while the latter quantity we now evaluate.

At a fixed multipole of the *true* density field ℓ_2 and a calibration error at some multipole ℓ_1 , only the *observed* multipoles in the range $\max\{|\ell_2 - \ell_1|, 0\} \leq \ell \leq \ell_2 + \ell_1$ are affected; this is well known feature of the coupling of angular momenta in e.g. quantum mechanics. Therefore, for a fixed values of ℓ and m , the total number of (ℓ_2, m_2) pairs that can possibly lead to nonzero U coefficients is naively $N_{\ell_2, m_2} = \sum_{\ell=\ell_{\text{calib,max}}}^{\ell+\ell_{\text{calib,max}}} (2\ell + 1)$. However, we must also remember of a selection rule⁹ that $m_1 + m_2 = m$, which means that for a fixed observed-field coefficient m and calibration-field coefficient m_1 , only *one* coefficient m_2 (instead of $2\ell_2 + 1$ of them) survives. Therefore, for a fixed ℓ , the total number of U coefficients to tabulate (e.g. for use in Eq. (14)), is of order

$$\begin{aligned} N_{m, \ell_2, m_2, \ell_1, m_1} &\simeq (2\ell + 1)(2\ell_{\text{calib,max}} + 1)N_{\ell_1, m_1} \\ &\simeq 4\ell \ell_{\text{calib,max}}^3 \end{aligned} \quad (\text{D2})$$

where the factors of $2\ell + 1$ and $2\ell_{\text{calib,max}} + 1$ refer to the number of m coefficients and (ℓ_2, m_2) pairs, respectively. So for a typical calculation with $\ell_{\text{calib,max}} \simeq 20$, the number of evaluations for a single multipole $\ell \sim 250$ is of order 10^7 ; for of order 30 values of ℓ , including the densely spaced coverage of lowest observable multipoles where the effects of photometric variation are the largest, this is still less than $O(10^9)$ evaluations of the Wigner-3j symbols, which is entirely feasible and takes of order a minute on a single 16-core desktop computer.

The last detail is actual computation of the Wigner-3j symbols: we use the **Gnu Scientific Library** (GSL) routines at $\ell \geq 50$. At higher multipoles these break down, and we use a special-purpose routine¹⁰ that uses approximate expressions for the Wigner coefficients. These expressions hold provided $\ell, \ell_2 \gg \ell_1$, but given that $\ell > 50$ in the regime we are using this approximation and we only consider $\ell_1 \leq \ell_{\text{calib,max}} \simeq 20$, this condition always holds.

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